Statistics of spiking neural networks based on counting processes

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Estimating neuronal network activity as point processes is challenging due to the singular nature of events and high signal dimensionality [1]. This project analyzes spiking neural networks (SNNs) using counting process statistics, which are equivalent integral representations of point processes [2]. A small SNN of Leaky Integrate-and-Fire (LIF) neurons is simulated, and spiking events are counted as a vector counting process N(t). The Poisson counting process has known dynamic statistics over time: both mean(t) and variance(t) are proportional to time (= $r_i(t)$ for each independent source with rate r_i). By standardizing the data, trivial mean dynamics and heteroscedasticity can be removed, allowing comparison against a baseline Poisson counting process.

A spiking neural network (SNN) with LIF neurons and Poisson inputs was simulated in NEST [3], generating spike trains for analysis. The counting process, a stochastic process producing the number of events within a time period, is analyzed using the vector counting process. Mean and covariance of spiking events are estimated for both SNN and Poisson processes, facilitating comparison of statistical properties after standardization by subtracting the mean and scaling by the standard deviation to account for temporal dependencies.

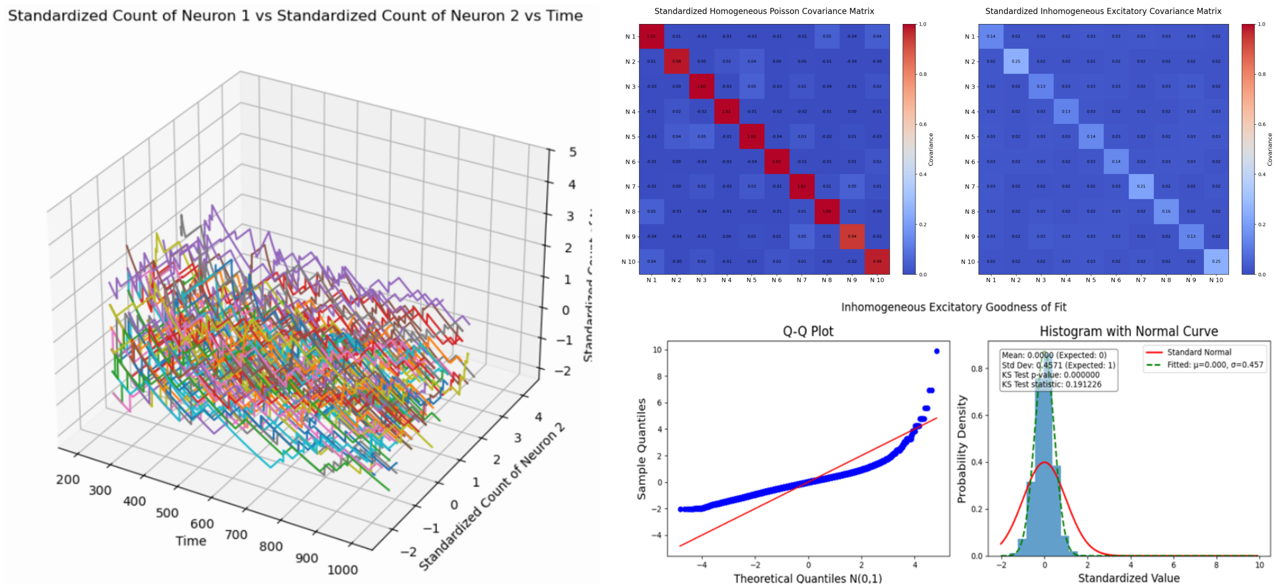

Figure 1 illustrates the standardized spiking dynamics of many trials of two neurons (out of many) over time. The standardized counts indicate variability aligned with SNN's statistical properties. Typical statistical items like mean, covariance (10 units out of many) and Q-Q plots are easily estimated with such data as contrasted to point process data. The covariance matrix quantifies relationships between neurons at certain time and activity levels. Comparing the SNN to modeled Poisson processes reveals notable differences in covariance structures, with the SNN demonstrating reduced auto- and greater inter-unit correlation. That is also evident in the Q-Q plot against equivalent Poisson model and histogram, highlighting lower SNN variance compared to Poisson.

1. Brown et al. Nature Neuroscience, 7(5), 456-461, 2004
2. Cox and Isham, Point processes. Chapman and Hall, 1980
3. Terhorst et al. NEST 3.9, Zenodo, 2025