On polynomials in spiking neuronal network simulations

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Exponentially decaying terms play a dominant roles in computational neuroscience. Rotter & Diesmann [1] show that these can often be expressed as the solutions of linear time-invariant (LTI) systems of differential equations, and therefore be integrated exactly on a fixed time grid. Polynomials, as already stated in the 1999 work, also fall into this class. Simple examples are box-like and parabolic synaptic currents. However, in contrast to the common exponentially decaying terms, polynomials generally grow unbounded.
LTI systems are ideal for spiking neuronal network simulations because they maintain the full history of past events in their state variables. Morrison et al. [2] solve the remaining problem of representing the initial conditions for frequently incoming events with different delays by a ring buffer.
Here, we show that polynomials over a fixed time interval fit into the framework by a generalization of the ring buffer. Next to the initial condition for the up kick, an incoming event just simultaneously needs to place a corresponding down kick on the ring buffer at the desired time segment in the future. Furthermore, we realize that from this point of view absolute refractoriness is a time period initiated by an event a neuron sends to itself.
While the practical impact of these insights may be limited, they help teaching the concept of concentrating non-linearities in neuron models into the boundary conditions of LTI systems.

[1] Rotter S, Diesmann M (1999) Exact digital simulation of time-invariant linear systems with applications to neuronal modeling. Biological Cybernetics 81(5-6):381-402

[2] Morrison A, Mehring C, Geisel T, Aertsen A, Diesmann M (2005) Advancing the Boundaries of High-Connectivity Network Simulation with Distributed Computing. Neural Computation 17(8):1776-1801