Energy Minimization and Multi-Stability in PLL-Based Coupled Oscillator Networks for Neuromorphic Computing

M. Shamookh(1), A. Ashok(1), S. Y. Neyaz(1), C. Wang(1), A. Zambanini(1), S. van Waasen(1,2)

(1) Peter Grünberg Institute - Integrated Computing Architectures (ICA|PGI-4), Forschungszentrum Jülich, Germany.

(2) Faculty of Engineering, Communication Systems, University of Duisburg-Essen, Duisburg, Germany.

Traditional computing suffers from high latency and energy consumption due to constant memory access, known as the Von Neumann bottleneck for AI. By contrast, brain waves, once dismissed as background noise, encode meaningful information through coordinated spatiotemporal activity [1]. Oscillatory neural networks (ONNs) exploit this by modeling neurons as coupled oscillators, enabling parallel, energy-efficient computation via synchronization, as described by Kuramoto dynamics [2]. However, scalable hardware remains challenging due to complex coupling networks, device mismatches and physical constraints, highlighting for efficient CMOS-based implementations, such as phase-locked loop (PLL) with Kuramoto-like phase–frequency dynamics, to realize the full potential of ONNs.

Within ONNs, coupled oscillators naturally evolve toward energy minimization, reflecting synchronization [2]. To capture this at the circuit level, a mechanical analogy for the PLL is established using a Lyapunov formulation, defining potential and kinetic energy based on phase difference and its rate of change. The phase detector (PD) shapes the energy landscape, determining the number and stability of energy wells as shown in Fig. 1a. In coupled PLLs, this extends to a multi-dimensional energy wells, where the global minima corresponds to full frequency and phase locking, whereas local minima get stable under specific external effects. This framework is realized in PLL-based coupled-oscillator (COOP) system shown in Fig. 1b [3], enabling hardware exploration of synchronization governed by PD characteristics.

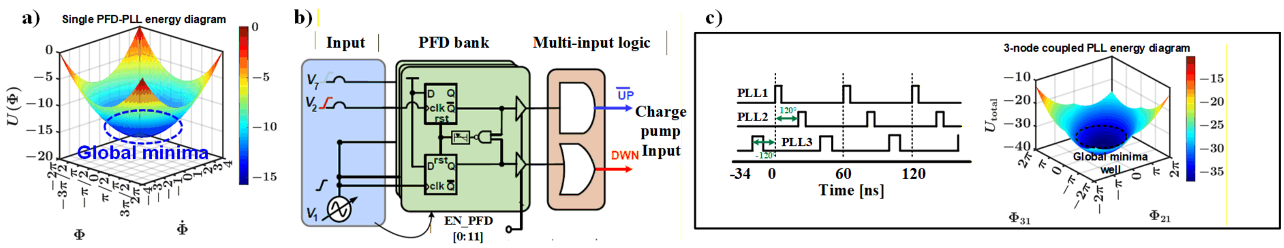

Figure 1: a) Phase frequency detector (PFD) energy well analytically developed using mechanical analogy. b) PLL-based coupled-osillator (COOP) implementation block diagram. c) 3-node graph (left) experimental measurements under external effect (right) analytical combined-network energy well.

For monotonic PDs such as phase frequency detector (PFD), multiple stable energy wells arise in coupled networks due to its constant positive slope, enforcing a zero-sum phase constraint, whereas non-monotonic PDs like mixers yield fewer stable minima due to destabilizing negative slope regions. This highlights that PD characteristics define the energy landscape and synchronization stability. These multi-stable energy wells are experimentally validated on a COOP system [3] across 3–5 node, networks shown in Fig. 1c for 3-node experimental result and total energy landscape, confirming ONN energy-minimization behavior in physical circuits and establishing PLL dynamics as a promising foundation for scalable COOP-based ONN architectures.

[1] Mendoza-Halliday, D. et al., Nat Neurosci 27, 547–560, 2024.

[2] Todri-Sanial, A. et al, npj Unconv. Comput. 1, 14, 2024.

[3] Neyaz, S. Y. et al., arXiv, 2505.10248, 2025.