

Towards Design of Bio-realistic third-order Memristive Neuron Circuits for Neuromorphic Computing

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As conventional computing architectures approach fundamental limits imposed by device scaling and the von Neumann bottleneck, neuromorphic computing has gained increased attention inspired by the remarkable computational and energy efficiency of biological neural systems across diverse cognitive tasks where computation emerges from continuous nonlinear dynamics and event-driven spiking activity. Biological neurons are inherently nonlinear dynamical systems that exhibit a wide range of behaviors, which are captured well by computational neuron models. Locally active memristors (LAMs) that exhibit negative differentiable resistance (NDR) enable natural hardware implementation of neuronal dynamics. However, existing designs of memristor-based neuron circuits [1-3] are dynamically different from their biological counterparts, as is evident from their subthreshold dynamics and spike waveforms. Furthermore, neuron-like behaviors are often demonstrated through empirical parameter tuning, lacking a clear connection to biological models.

In this work, we develop a principled bottom-up approach for the design of bio-realistic third-order memristor-based neuron circuits, through a dynamical systems perspective based on phase flow analysis and bifurcation diagrams. We begin with the standard second-order LAM neuron circuit and show that replacing the linear bias resistor with a nonlinear resistor exhibiting diode-like characteristics reproduces bio-plausible nullclines and dynamics. Through bifurcation analysis, we identify five operating regions, including two novel bistable regimes B_1 and B_2 which arise from a series of co-existing saddle-node and supercritical Hopf bifurcations. Regime B_1 contains a bistability between two stable fixed points, whereas B_2 contains coexistence of a stable fixed point and a limit cycle, enabling switching via initial conditions or transient pulses between UP/DOWN states in B_1 and between quiescence and spiking in B_2 . To the best of our knowledge, such a rich bifurcation structure has not been demonstrated in a physically grounded electrical neuron circuit before.

Furthermore, this second-order system is then extended to a third-order memristive neuron circuit where the bistable regimes B_1 and B_2 are exploited to exhibit bursting and adaptation dynamics through fast-slow decomposition. The third state variable is introduced by incorporating threshold-based slow dynamics into the nonlinear resistor, transforming it into a nonlinear memristive element. Thus, the third-order system arises from slow modulation of the second-order dynamics, analogous to fast-slow biological neuron models. It is important to note that the circuit function is agnostic to the exact form of the modulation. We analyze the bursting mechanism and show that it corresponds to Hopf/Fold geometry of bursting. Finally, we analyze the I-V characteristics of the nonlinear memristor to ensure that it is physically grounded and show that it features volatile inverse hysteresis characteristics similar to that of the biological potassium ion channel [4]. This highlights the importance of inverse hysteresis as a key device property for constructing memristive neuron circuits that are truly analogous to biological neuron models, offering new opportunities and motivation for engineering of such devices.

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- [3] Kumar et al., *Nature*, 585(7826):518–523, 2020.
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