

A Generalized Bifurcation Analysis Framework for Spiking Neuron Models

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Neuromorphic computing (NC) targets energy-efficient event-driven information processing by placing spiking-neurons at its core [1]. At the hardware level, NC systems require sophisticated design and optimization of neurons at device and circuit levels. Neuron devices and circuits have abundant nonlinear dynamics that cannot be addressed by analysis methods for linear systems [2,3]. Here, we present a generalized bifurcation analysis framework to evaluate the nonlinear behavior of neurons and, furthermore, to guide the design and optimization of artificial neurons. Methodologically, the framework is based on dynamical systems theory, including Hopf bifurcation, and is formulated in a generalized manner that does not rely on any particular circuit structure or artificial neuron realization. Here, the CMOS axon-hillock neuron is employed as an example to demonstrate the analysis by the proposed framework in phase portrait (Fig. 1(a,b)). During the design and optimization, it is critical to select a group of parameters that make the neuron fire. Following the framework, such a firing domain in the parameter space is defined by the conditions from the Hopf bifurcation and the operating region boundaries (Fig. 1(c)). In other words, all the parameter combinations in the firing domain will guarantee a firing neuron. Moreover, the firing rate at the selected points in the firing domain is evaluated (Fig. 1(d)). The firing rate obtained from the framework is aligned well with that from a numerical simulation of the system, indicating the validity of the framework (Fig. 1(e)). To demonstrate the capability of the generalization, the framework is also applied to a memristor neuron (Fig. 1(f)). The framework standardizes the analysis for various neuron models and physical realizations, which offers practical design and optimization guidelines for artificial neurons in NC systems.

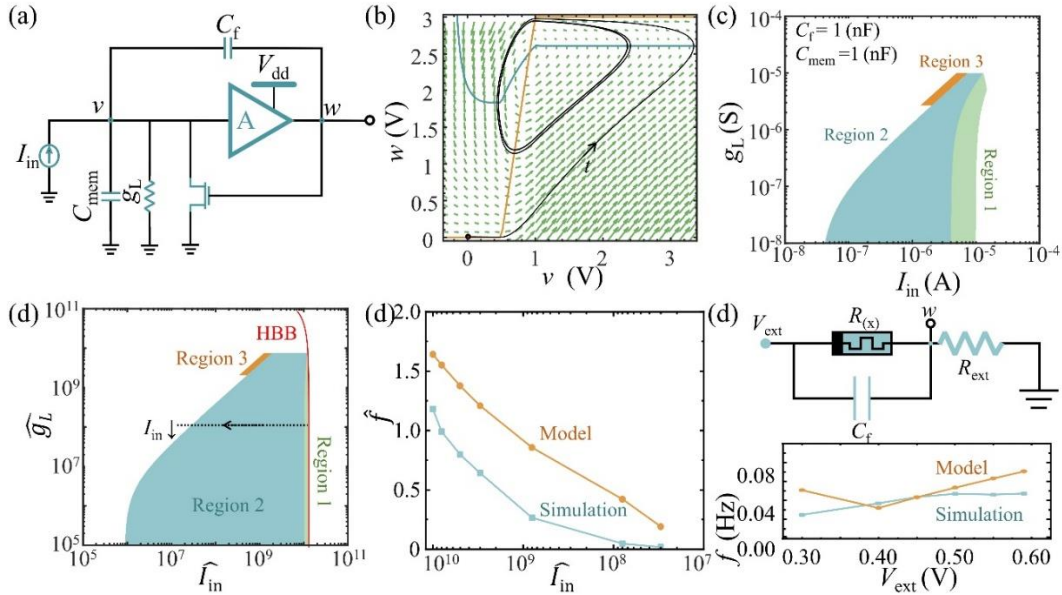


Figure 1: Generalized bifurcation analysis of spiking neuron models.

[1] J. Pei et al., Nature, vol. 572, no. 7767, pp. 106–111, 2019.

[2] M. Xiao et al., IEEE TNNLS, vol. 32, no. 5, pp. 1974-1988, 2021.

[3] M. Yao et al. Nat Commun 15, 4464, 2024.