Stabilizing Linear Systems with Spiking Neural Networks

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Neuromorphic control is an emerging field that aims to mimic the workings of biological neurons to harness their advantages such as low energy consumption, quick event-based responses, adaptability and robustness in decision-making and control systems. The key difference with conventional, typically time-driven controllers is the event-based nature where neurons fire as a response to an event and communicate through very short pulses, called spikes. Control design methods already exist for either event-based or impulsive control but there is a lack of design methods for controllers that are both event-based and impulsive. In this work, this topic is investigated by constraining the control input to a train of spikes which are modeled as Dirac pulses with a fixed amplitude. Note that the only remaining control freedom is the mechanism that regulates the timing of when to fire which spike. We present systematic methods for the design of such control mechanisms leading to the practical stabilization of linear time-invariant (LTI) systems controlled by such spikes. See also [1] and [2].
Consider a stabilizable LTI system which is to be controlled by bidirectional Dirac spikes

$$\dot{x}(t) = Ax(t) + Bu(t), \quad u_k(t) = \sum_i \alpha_k \delta(t - t_{k,i}^+) - \sum_j \alpha_k \delta(t - t_{k,j}^-) \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}, u(t) = (u_1(t), u_2(t), \ldots, u_{n_u}(t))^\top \in \mathbb{R}^{n_u}, y(t) \in \mathbb{R}^{n_y}, t \in \mathbb{R}_{\geq 0}$, α_k is the amplitude of the spike on the k-th input channel, $t_{k,i}^+$ and $t_{k,j}^-$ are, respectively, the positive and negative spiking times and $k \in \{1, 2, \ldots, n_u\}$. Note that we idealized the spike as a Dirac function for modelling convenience. The system is in open loop between spikes and instantly changes state as a result of the spikes. The goal is to formulate a spiking strategy to determine these spiking times such that the state (practically) asymptotically converges to the origin.
The idea for the spiking strategy is inspired by a recent paper [3], but there, a formal analysis was lacking. This work generalizes and extends [3] with a mathematical analysis and guarantees for the designed controllers. The key idea is to first choose an appropriate quadratic Lyapunov function $V(e) := x^\top Px$ with P a positive definite matrix. The spiking condition is then chosen as

$$V(x \pm \alpha_k B_k) + c_k \leq V(x), \tag{2}$$

such that the Lyapunov function always decreases by at least $c_k > 0$ when a spike is fired in the k-th input channel. Here $B = (B_1, B_2, \ldots, B_{n_u})$. It follows that there exists a level set of the Lyapunov function ($V(x) \leq d$) for which $\dot{V} \leq 0$ if V is chosen appropriately. Since the Lyapunov function also decreases at spikes, this set is an attractor around the origin. Therefore, global practical stabilization can be achieved, i.e. the state x converges to a neighborhood of the origin as time goes to infinity.
This method is further extended to obtain a controller in the form of a spiking neural network (SNN). This method is proven to globally practically stabilize the system while always having a positive inter-spike time in the single-input case, see [2].
Future work will focus on extending this method to nonlinear systems and guaranteeing a positive inter-spike time also in the multi-input case.

[1] W.P.M.H. Heemels et al., "Spiking Control for Stabilization and Tracking in Linear Systems: A Greedy Lyapunov-based Approach", ECC 2026.

[2] W.M. Klip et al., "Stabilizing Linear Time-Invariant Systems with Recurrent Spiking Neural Networks", under review.

[3] P. Agliati et al., "Spiking neurons as predictive controllers of linear systems", arXiv preprint arXiv:2507.16495, 2025.